Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-09-20

Conceptual questions

Question 1. Suppose $f(x, y)$ is a function and $P \in \mathbb{R}^2$ is a point such that $\nabla f(P) = \langle 5, 12\rangle$. What number should be put in the blank below

$$\langle 5, 12, \_\_\_\rangle$$

to get a normal vector for the tangent plane to $z = f(x, y)$ at the point P? How about a vector parallel to that plane?

Question 2. Suppose $g(x, y)$ is a function, $\mathbf{u} = \langle 3/5, 4/5\rangle$ and $Q \in \mathbb{R}^2$ is a point such that $D_{\mathbf{u}}g(Q) = -3$. What number should be put in the blank below

$$\left\langle \frac{3}{5}, \frac{4}{5}, \_\_\right\rangle$$

to get a vector parallel to the tangent plane to $z = g(x, y)$ at the point Q? Is there enough information to obtain a normal vector? What if you were also told that $D_{\mathbf{v}}g(Q) = 4$, where $\mathbf{v} = \langle 4/5, 3/5\rangle$?

Computations

Problem 1. This problem is recycled from the last worksheet (we hadn't covered the chain rule at that time).

(a) Suppose that $\mathbf{r}(t) = (x(t), y(t))$ is parametrized by arclength (recall that this means $|\mathbf{r}'(t)| = 1$; the particle is "moving at speed 1"). Show that the directional derivative of f in the direction of $\mathbf{r}'(t)$ is equal to $\frac{\mathrm{d}}{\mathrm{d}t}(f(\mathbf{r}(t)))$. Hint: Use the chain rule.

(b) Consider the function

$$f(x, y) = \cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right).$$

Using the preceding part, compute $f_y(1, 0)$. Hint: Use the unit circle. You should not need the derivative of arccosine.